**Matrix and Determinant**

**1.** Use the Crammer’s rule to discuss the consistency of the following system of equation for different cases of λ:

 $\left\{\begin{array}{c}x+y+λz=1\\x+λy+z=λ\\λx+y+z=λ^{2}\end{array}\right.$

**2.** Let A =  , B =  where α ≠ 1.

 **(a)** Prove that for all positive integer n,

 

 **(b) (i)** Find BA.

**(ii)** Use **(a)** , or otherwise, evaluate (BA) n , for n ∈ **N,**

**3.** **(a)** Factorize .

 **(b)** Show that  = k F(x,y,z) F(a,b,c)

 where k is a constant to be found, and hence factorize the determinant.

**4.** If n is the least positive integer such that An is a zero matrix, then A is said to be nilpotent of order n.

 Given A is a nilpotent of order n.

 **(a) (i)** Evaluate  and

  

  **(ii)** Hence, or otherwise, express (I – A) -1 and (I – A2)-1 in terms of A.

 **(b)** Let A = 

**(i)** Evaluate A2 and A3.

 **(ii)** Using (a), or otherwise, find (I – A) -1 and (I – A2)-1.

**5.** Let A = 

**(a)** Find A3 – 2A2 – 7A + I where I is the identity matrix of order 3 x 3.

**(b)** Using (a), evaluate (A – I ) ( A2 – A – 8 I ).

**(c)** Hence find (A2 – A – 8 I ) -1.

**6.** Find the equation of the image of the curve :

 

 if the **curve** is under the rotation transformation through an angle  anti-clockwisely about the origin.  **Hint :** The formula for rotating anti-clockwisely by an angle θ is

 $\left(\begin{matrix}x^{'}\\y^{'}\end{matrix}\right)=\left(\begin{matrix}\cos(θ)&-\sin(θ)\\\sin(θ)&\cos(θ)\end{matrix}\right)\left(\begin{matrix}x\\y\end{matrix}\right)$

**1. **



 **Conclusions :**

 **(i)** If λ ≠ 1, – 2, Δ ≠ 0. ∴ The system has unique solution:

 .

 **(ii)** If λ = 1, Δ = Δx = Δy = Δz and the system of equation becomes x + y + z = 1 .

 ∴ The system of equation has infinitely many solutions:

 (x , y , z) = (λ1 , λ2 , 1 – λ1 – λ2) .

 **(iii)** If λ = – 2, Δ = 0 , Δx ≠ 0, Δy ≠ 0, Δz ≠ 0.

 ∴ The system is inconsistent and has no solution.

**2. (a)** Let P(n) be the proposition: 

 For P(1), . ∴ P(1) is true.

 Assume P(k) is true for some k ∈ N, i.e.  (1)

 For P(k + 1), Ak+1 = Ak ⋅ A =  , by (1).

 

 .

 ∴ P(k + 1) is true.

 **(b)**  ∴ 

**3. (a)** 

 

 **(b) Method 1**



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 **=** 2 F(x, y, z) F(a, b, c)

 **=** 2(x – y) (y – z) (z – x) (a – b) (b – c) (c – a)

 **Method 2**

Let Δ be the given determinant.

 Put x = y in Δ, Since R1 = R2 , Δ = 0 and (x – y) is a factor of Δ.

 Put y = z in Δ, Since R2 = R3 , Δ = 0 and (y – z) is a factor of Δ.

Put z = x in Δ, Since R3 = R1 , Δ = 0 and (z – x) is a factor of Δ.

 Put a = b in Δ, Since C1 = C2 , Δ = 0 and (a – b) is a factor of Δ.

 Put b = c in Δ, Since C2 = C3 , Δ = 0 and (b – c) is a factor of Δ.

Put c = a in Δ, Since C3 = C1 , Δ = 0 and (c – a) is a factor of Δ.

∴ Δ = kF(x, y, z) F(a, b, c) = k(x – y) (y – z) (z – x) (a – b) (b – c) (c – a).

**4. (a) (i)** (I – A)(I + A + A2 + … + An–1 = I – An = I – **0** = I (since An = **0**)

 = I + (– 1)n An = I – **0** = I (since An = **0**)

 **(ii)** (I – A)-1 = I + A + A2 + … + An–1

 (I + A)-1 = I – A + A2 – … + (–1)n–1An–1

 (I – A2)-1 = (I + A + A2 + … + An–1) [I – A + A2 – … + (–1)n–1An–1]

 = I + A2 + A4 + … + A2(n–1)

 

 **(b) (i)** 

  **(ii)** By (b) (i), A is a nilpotent matrix of order 3.

 By (a) (ii), (I – A)-1 = I + A + A2

 

 By (a) (ii), (I – A2)-1 = I + A2

 

**5. (a)** 

 ∴ A3 – 2A2 – 7A + I

 

 **(b)** (A – I) (A2 – A – 8I) = A3 – 2A2 – 7A + 8I = (A3 – 2A2 – 7A + I) + 7I , by (a).

 = 

 **(c)** By (b), (A2 – A – 8I)-1 = 

**6.** The matrix of rotation = 

 

 ∴ 

 Equation of the image is:

 

 Simplify, we get 16(x’)2 + 32 (y’)2 – 16 = 0

 (x’)2 + 2(y’)2 = 1 , an ellipse.

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